

$$X^*, Y^*, Z^* = \underset{X, Y, Z}{\operatorname{argmax}} P(X, Y, Z)$$

$$\underline{P(X_1, X_2, \dots, X_n)}$$

~~max~~

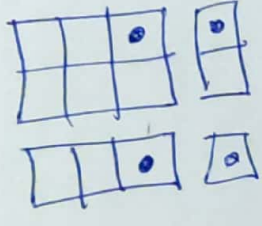
$$h(X, Y) = \max_Z P(X, Y, Z)$$

$$g(X) = \max_{Y, Z} P(X, Y, Z)$$

$$= \max_Y h(X, Y)$$

$$= \max_Y \max_Z P(X, Y, Z)$$

$$= \max_Z \max_Y P(X, Y, Z)$$



$$\underline{X^*, Y^*, Z^* = \underset{X, Y, Z}{\operatorname{argmax}} P(X, Y, Z)}$$

$$g(X) = \max_Y \max_Z P(X, Y, Z)$$

$$X^{**} = \underset{X}{\operatorname{argmax}} g(X)$$

$$X^{**} = P^*$$

if X^*, Y^*, Z^* unique

$$P(X^*, Y^*, Z^*) = \max_X \max_Y \max_Z P(X, Y, Z) = P^*$$

$$g(X^{**}) = \max_X g(X) = \max_X \max_Y \max_Z P(X, Y, Z) = P^*$$

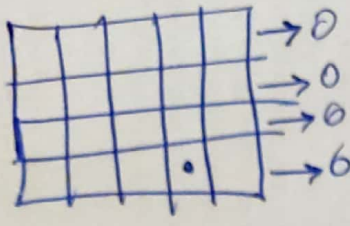
$$P^* = g(X^{**}) = \max_Y \max_Z P(X^{**}, Y, Z) = P(X^{**}, Y^{**}, Z^{**}) = P^*$$

$$P(X^{**}, Y^{**}, Z^{**}) = P(X^*, Y^*, Z^*)$$

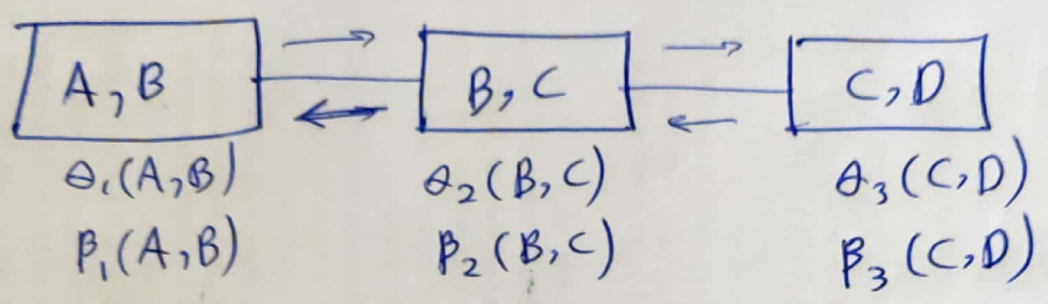
$$X^*, Y^*, Z^* \text{ unique} \Rightarrow X^{**} = X^*$$

$$P^{**} = P^*$$

$$\max_{X, Y} g(X, Y) = \max_X \underbrace{\max_Y g(X, Y)}_{h(X)}$$



pgm 16 (E)



$$\theta(A, B, C, D) = \theta_1(A, B) \theta_2(B, C) \theta_3(C, D)$$

$$\boxed{0 \ 1 \ 0 \ 1} \\ A^*, B^*, C^*, D^* \in \operatorname{argmax} \theta(A, B, C, D)$$

$$\boxed{1 \ 0 \ 1 \ 0} \\ A^{**}, B^{**}, C^{**}, D^{**} \in \operatorname{argmax} \theta(A, B, C, D) \\ B^* \neq B^{**}$$

$$P(X) = P(X_1, X_2, \dots, X_n) = \frac{1}{Z} e^{-E(X)}$$

$$E(X) = E(X_1, X_2, \dots, X_n) = \sum_{i=1}^n E_{X_i}(X_i) + \sum_{(i,j) \in E} E_{X_i, X_j}(X_i, X_j)$$

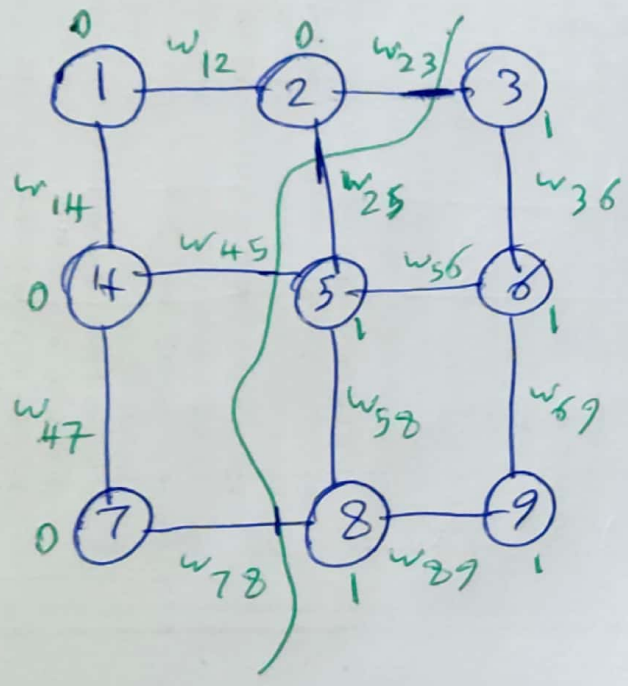
$$X_i \in \{0, 1\} \quad E_{X_i, X_j}(X_i, X_j) = w_{ij} \mathbb{1}(X_i \neq X_j)$$

$$E(X) = \sum_{i=1}^n$$

$$E(X) = \sum_{i=1}^n E_i(X_i) + \sum_{(i,j) \in E} E_{ij}(X_i, X_j)$$

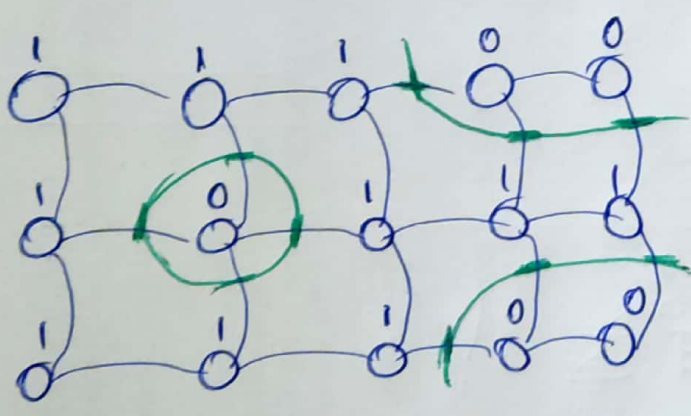
$$= \sum_{i=1}^n E_i(X_i) + \sum_{(i,j) \in E} w_{ij} \mathbb{1}(X_i \neq X_j)$$

$X_i \in \{0, 1\}$



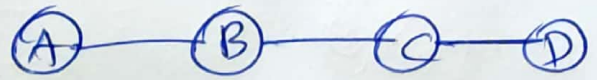
$$\text{cut} = w_{23} + w_{25} + w_{45} + w_{78}$$

$X_0 \ X_1 \ X_2$
 $\text{cut}(0, 0, 1, 0, 1, 1, 0, 1, 1)$
 $= w_{23} + w_{25} + w_{45} + w_{78}$

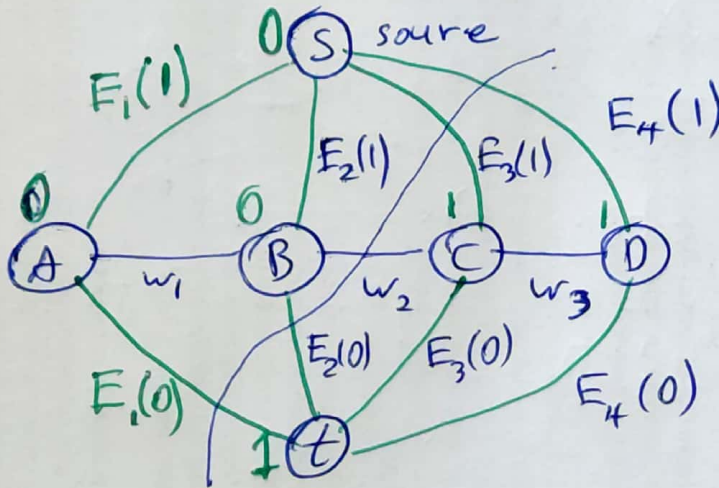


$$E(X) = \sum_{(i,j) \in E} w_{ij} \mathbb{1}(X_i \neq X_j)$$

$$E(X_1, X_2, X_3, X_4)$$



$$E(A, B, C, D) = E_1(A) + E_2(B) + E_3(C) + E_4(D) \\ + w_1 \mathbb{1}(A \neq B) + w_2 \mathbb{1}(B \neq C) + w_3 \mathbb{1}(C \neq D)$$



$$\text{Cut}(A, B, C, D) = E(A, B, C, D)$$

Special case $E(X_i, X_j) = \sum w_{ij} \mathbb{1}(X_i \neq X_j)$

$$\underline{E_{ij}(X_i, X_j)}$$

$$E_{ij}(0,0) + E_{ij}(1,1) \leq E_{ij}(1,0) + E_{ij}(0,1)$$

for all $(i,j) \in E$

Submodular